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# Symmetric and Probabilistic Quantum State Sharing via Positive Operator-valued Measure

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**Abstract** By using positive operator-valued measure, I present a new tripartite scheme for realizing quantum state sharing of an arbitrary unknown single-qubit pure state in either distant agent's place. In this scheme, the sender averagely partitions its unknown single-qubit state with a Greenberger-Horne-Zeilinger (GHZ)-basis measurement. Then by performing a proper positive operator-valued measure, one agent can recover the splitter's state in a probabilistic manner provided that he/she gets another one's help. Moreover, the total success probability of the scheme is also worked out. Finally, I concisely generalize the tripartite QSTS scheme to a multiparty case.

**Keywords** Quantum state sharing · Projective measurement · Greenberger-Horne-Zeilinger (GHZ)-basis measurement · Positive operator-valued measure

## 1 Introduction

With the theory of quantum mechanics in the field of information, many interesting developments have been produced in last decades, such as quantum key distribution [1, 2], quantum teleportation [3, 4], quantum dense coding [5, 6], quantum secret sharing (QSS) [7–11], etc. [12–14]. QSS is the quantum counterpart of classical secret sharing [7, 8]. This generalization from classical information processing to quantum information processing was first presented by Hillery, Bŭzek and Berthiaume (HBB) in 1999 [9]. The basic idea of QSS in the simplest case can be described as follows. The initial owner splits a secret message into two pieces via quantum mechanical method and sends each sharer a piece so that neither of the two sharers is able to obtain the secret information unless they cooperate together. QSS is likely to play a key role in both transmitting of classical message and protecting secret quantum information, e.g., in secure operations of distributed quantum computation, sharing difficult-to-construct ancilla states and joint sharing of quantum money [15], and so on.

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By far it has attracted much attention and many QSS schemes have already been proposed [16–30].

The QSS of quantum information is conventionally termed as quantum state sharing (QSTS) [11] to differentiate from the QSS of classical message. Up to now, different quantum channels consisting of various entangled states have been explored in QSTS, such as GHZ states [31–34], W states [35–37], Graph states [38], Bell states [39–41], etc. Recently, people focus more attentions on the non-maximally entangled states as quantum channels in QSTS schemes [42–44]. In this case, one or more auxiliary qubits need to be incorporated and entangled with the original qubits. After this, a proper measurement on qubits including the ancillas should be executed such that the original-qubit state is collapsed to one of the eligible states. Subsequently, the original state is retrieved from the eligible state by performing an appropriate unitary operations which correspond to the measurement outcomes. Note that, the so-called proper measurement are usually projective measurement in the latter type of existing QSTS schemes [42–44]. As a matter of fact, there lies another type of measurement named positive operator-valued measure (POVM) [45, 46]. Very recently it has already attracted many attentions and been employed in various quantum information processing [45–49]. Nonetheless, to our best knowledge, so far it has never been introduced in QSTS. In this paper, by using POVM instead of usual projective measurement, I will propose a new scheme for sharing an arbitrary unknown *single-qubit* pure state with two non-maximally entangled two-qubit states.

This paper is organized as follows. In Sect. 2, a symmetric tripartite scheme for probabilistically sharing an arbitrarily unknown single-qubit pure state is amply presented by utilizing POVM. In Sect. 3, the generalization of the tripartite QSTS scheme to a multiparty case is sketched. Finally, some remarks and summaries are given in Sect. 4.

## 2 Tripartite QSTS Scheme via POVM

Now let me amply introduce the tripartite QSTS scheme for sharing an arbitrary unknown single-qubit quantum state with two non-maximally entangled 2-qubit states. The schematic demonstration is illustrated in Fig. 1. As shown, the scheme contains three legitimate parties. Suppose they are Alice, Bob and Charlie. Alice is the boss (also the sender) in the scheme, Bob and Charlie are her two agents. Alice has an unknown quantum state

$$|P\rangle_x = \alpha|0\rangle_x + \beta|1\rangle_x, \tag{1}$$

where $\alpha$ and $\beta$ are unknown complex numbers and satisfy $|\alpha|^2 + |\beta|^2 = 1$. Moreover, Alice, Bob and Charlie share two non-maximally entangled 2-qubit states. See Fig. 1(a). Qubit pair (1, 3) belongs to Alice, Bob holds qubit 2, and qubit 4 is at Charlie's place. Without loss of generality their states read

$$|\psi\rangle_{12} = a|00\rangle_{12} + b|11\rangle_{12}, \qquad |\phi\rangle_{34} = c|00\rangle_{34} + d|11\rangle_{34}, \tag{2}$$

where $a$, $b$, $c$ and $d$ are nonzero real numbers, they satisfy $|a| \geq |b|$, $|c| \geq |d|$, $|a|^2 + |b|^2 = 1$ and $|c|^2 + |d|^2 = 1$, respectively. In this case, the combined state of the five qubits is

$$|\Phi\rangle_{x1234} = |P\rangle_x \otimes |\psi\rangle_{12} \otimes |\phi\rangle_{34}. \tag{3}$$

Alice wants to split her unknown state between Bob and Charlie by utilizing the quantum channels. About the split, her requirement is that neither agent can get the quantum state

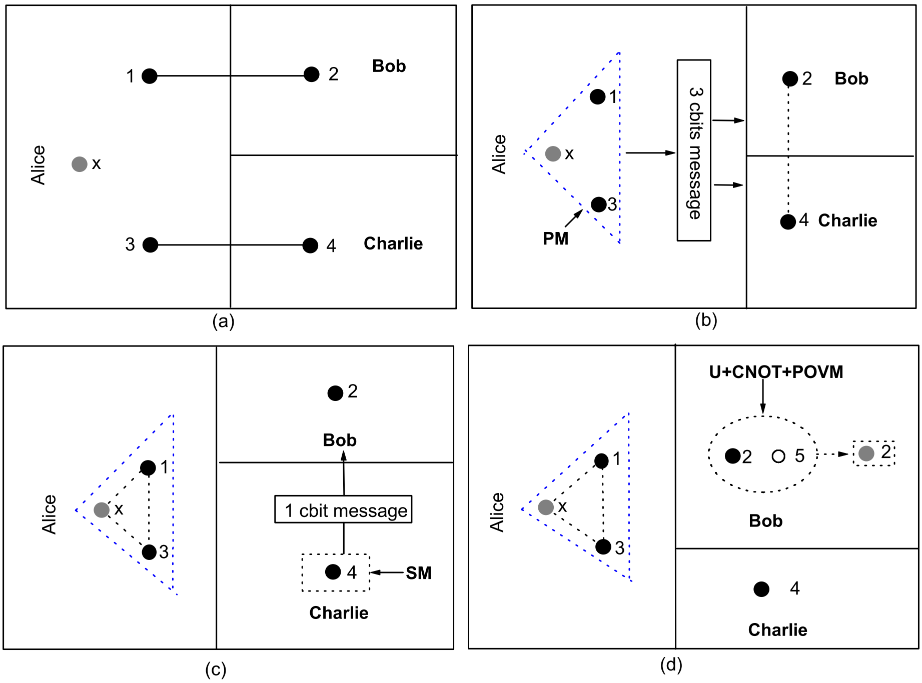


**Fig. 1** Partition of an unknown arbitrary single-qubit quantum state among 2 recipients by using two non-maximally entangled 2-qubit states. The *solid dots* represent qubits ($x$, 1–4), the *litter hollow circle* stands for the auxiliary qubit (5). The dotted triangle characterizes GHZ-basis projective measurement (PM). SM symbolizes a single-qubit measurement. CNOT means the controlled-not gate operation and POVM represents the positive operator-valued measure. See text for more details

solely unless they cooperate. To achieve her goal, Alice first measures the three qubits in her place in the basis set $\{\lambda_i\}, i = 1, 2, 3, \ldots, 8$ (see Fig. 1(b)), which forms a complete orthonormal basis of the combined Hilbert space of the three spin-$1/2$ particles (or two-level systems):

$$|\lambda_1\rangle = \frac{1}{\sqrt{2}}(|000\rangle + |111\rangle),$$

$$|\lambda_2\rangle = \frac{1}{\sqrt{2}}(|001\rangle + |110\rangle),$$

$$|\lambda_3\rangle = \frac{1}{\sqrt{2}}(|010\rangle + |101\rangle),$$

$$|\lambda_4\rangle = \frac{1}{\sqrt{2}}(|100\rangle + |011\rangle),$$

$$|\lambda_5\rangle = \frac{1}{\sqrt{2}}(|000\rangle - |111\rangle),$$

$$|\lambda_6\rangle = \frac{1}{\sqrt{2}}(|001\rangle - |110\rangle),$$

$$|\lambda_7\rangle = \frac{1}{\sqrt{2}}(|010\rangle - |101\rangle),$$

$$|\lambda_8\rangle = \frac{1}{\sqrt{2}}(|100\rangle - |011\rangle). \tag{4}$$

After Alice's measurement, the system's state collapses to one of the following eight possible results:

$$|\lambda_1\rangle_{x13}\langle\lambda_1|\Phi\rangle_{x1234} = \frac{1}{\sqrt{2}}|\lambda_1\rangle_{x13}(\alpha ac|00\rangle_{24} + \beta bd|11\rangle_{24}), \tag{5}$$

$$|\lambda_2\rangle_{x13}\langle\lambda_2|\Phi\rangle_{x1234} = \frac{1}{\sqrt{2}}|\lambda_2\rangle_{x13}(\alpha ad|01\rangle_{24} + \beta bc|10\rangle_{24}), \tag{6}$$

$$|\lambda_3\rangle_{x13}\langle\lambda_3|\Phi\rangle_{x1234} = \frac{1}{\sqrt{2}}|\lambda_3\rangle_{x13}(\alpha bc|10\rangle_{24} + \beta ad|01\rangle_{24}), \tag{7}$$

$$|\lambda_4\rangle_{x13}\langle\lambda_4|\Phi\rangle_{x1234} = \frac{1}{\sqrt{2}}|\lambda_4\rangle_{x13}(\beta ac|00\rangle_{24} + \alpha bd|11\rangle_{24}), \tag{8}$$

$$|\lambda_5\rangle_{x13}\langle\lambda_5|\Phi\rangle_{x1234} = \frac{1}{\sqrt{2}}|\lambda_5\rangle_{x13}(\alpha ac|00\rangle_{24} - \beta bd|11\rangle_{24}), \tag{9}$$

$$|\lambda_6\rangle_{x13}\langle\lambda_6|\Phi\rangle_{x1234} = \frac{1}{\sqrt{2}}|\lambda_6\rangle_{x13}(\alpha ad|01\rangle_{24} - \beta bc|10\rangle_{24}), \tag{10}$$

$$|\lambda_7\rangle_{x13}\langle\lambda_7|\Phi\rangle_{x1234} = \frac{1}{\sqrt{2}}|\lambda_7\rangle_{x13}(\alpha bc|10\rangle_{24} - \beta ad|01\rangle_{24}), \tag{11}$$

$$|\lambda_8\rangle_{x13}\langle\lambda_8|\Phi\rangle_{x1234} = \frac{1}{\sqrt{2}}|\lambda_8\rangle_{x13}(\beta ac|00\rangle_{24} - \alpha bd|11\rangle_{24}). \tag{12}$$

From above equations, one can see, the state of qubits 2 and 4 becomes a pure entangled two-qubit state after GHZ-basis measurement. In this way, now the quantum information is split to the pure entangled state which is shared between Bob and Charlie, the distribution of quantum information is completed.

Alice then tells Bob and Charlie her measurement result via a classical channel (see Fig. 1(b)). Although each outcome dose not occur with equal probability (because (5)–(12) are unnormalized, it can not be considered the probability of each outcome occurs is $\frac{1}{8}$ equally), the subsequence is similar. Consequently, I take one case as an example to show the QSTS process hereafter. Without loss of generality, suppose Alice's measurement result is $|\lambda_8\rangle_{x13}$. According to (12), the state of qubits 2 and 4 collapses to

$$|T\rangle_{24} = \frac{1}{\sqrt{2}}(\beta ac|00\rangle_{24} - \alpha bd|11\rangle_{24}). \tag{13}$$

As proposed above, each one, Bob or Charlie, has the chance to get the original state. Without loss of generality, suppose Bob is assigned to recover the original state $|P\rangle$. Then Charlie is asked to measure his qubit 4 in the $X$ direction. This measurement is a single-qubit projective measurement in a set of two mutually orthogonal basis vectors

$$|+\rangle = \frac{1}{\sqrt{2}}(|0\rangle + |1\rangle), \qquad |-\rangle = \frac{1}{\sqrt{2}}(|0\rangle - |1\rangle). \tag{14}$$

In this basis set, the joint state $|T\rangle_{24}$ can be rewritten as

$$\begin{aligned}|T\rangle_{24} &= \frac{1}{\sqrt{2}}(\beta ac|00\rangle_{24} - \alpha bd|11\rangle_{24}) \\ &= \frac{1}{2}[|+\rangle_4(\beta ac|0\rangle_2 - \alpha bd|1\rangle_2) + |-\rangle_4(\beta ac|0\rangle_2 + \alpha bd|1\rangle_2)]. \end{aligned} \tag{15}$$

After Charlie's single-qubit projective measurements, if his measurement results are $|+\rangle_4$, the state of Bob's qubit 2 collapses to

$$|K\rangle_2 = \frac{1}{2}(\beta ac|0\rangle_2 - \alpha bd|1\rangle_2). \tag{16}$$

At this stage, in order to recover the original state, Bob cooperates with Charlie to get his help. Provided Charlie agrees to cooperate with Bob, he will communicate his measurement result to Bob over a public channel (see Fig. 1(c)). After having received Charlie's classical message about his outcome, Bob can reconstruct the original state as follows (see Fig. 1(d)).

(a) Bob performs a local unitary operation $U_1 = |1\rangle\langle 0| - |0\rangle\langle 1|$ on his qubit 2, which transforms the collapsed state $|K\rangle_2$ into

$$|K'\rangle_2 = \frac{1}{2}(\alpha bd|0\rangle_2 + \beta ac|1\rangle_2). \tag{17}$$

(b) Bob introduces an auxiliary qubit $m$ in the state $|0\rangle$. After the incorporation, Bob performs a controlled-not (CNOT) gate operation on the auxiliary qubit $m$ with qubit 2 as the controlled qubit. The CNOT operation $C_{2m}$ converts the state of the qubits 2 and $m$ to

$$\begin{aligned}|W\rangle_{2m} = C_{2m}|K'\rangle_2|0\rangle_m &= \frac{1}{2}(\alpha bd|00\rangle_{2m} + \beta ac|11\rangle_{2m}) \\ &= \frac{1}{4}[|E\rangle_2 \otimes |F\rangle_m + |G\rangle_2 \otimes |H\rangle_m], \end{aligned} \tag{18}$$

where

$$\begin{aligned} |E\rangle_2 &= \alpha|0\rangle_2 + \beta|1\rangle_2, \qquad & |F\rangle_m &= bd|0\rangle_m + ac|1\rangle_m, \\ |G\rangle_2 &= \alpha|0\rangle_2 - \beta|1\rangle_2, \qquad & |H\rangle_m &= bd|0\rangle_m - ac|1\rangle_m. \end{aligned}$$

From (18), one can see that, Bob can get the state $|E\rangle_2$ or $|G\rangle_2$ provided that the states $|F\rangle_m$ and $|H\rangle_m$ are distinguished via an appropriate measurement. Note that $|E\rangle_2$ is exactly the original state. Readily, the original state can be further retrieved from $|G\rangle_2$. Unfortunately, the states $|F\rangle_m$ and $|H\rangle_m$ are not orthogonal in general. As a consequence, they can not be differentiated deterministically. In order to discriminate the two states with a certain probability, Bob adopts to perform an optimal POVM measurement [45–47] on the auxiliary qubit $m$. The POVM takes the form as

$$O_1 = \frac{1}{\omega}|M_1\rangle\langle M_1|, \qquad O_2 = \frac{1}{\omega}|M_2\rangle\langle M_2|, \qquad O_3 = I - \frac{1}{\omega}\sum_{i=1}^{2}|M_i\rangle\langle M_i|, \tag{19}$$

where

$$|M_1\rangle = \frac{1}{\sqrt{\xi}}\left(\frac{1}{bd}|0\rangle + \frac{1}{ac}|1\rangle\right)_m, \qquad |M_2\rangle = \frac{1}{\sqrt{\xi}}\left(\frac{1}{bd}|0\rangle - \frac{1}{ac}|1\rangle\right)_m,$$

$$\xi = \frac{1}{(bd)^2} + \frac{1}{(ac)^2} = \frac{1-b^2-d^2+2b^2d^2}{(1-b^2)(1-d^2)b^2d^2},$$

$I$ is an identity operator, and the parameter $\omega$ relating to $a$, $b$, $c$ and $d$ should assure $O_3$ positive. To determine upper and lower limits of the parameter $\omega$, the three operators $O_1$, $O_2$ and $O_3$ are rewritten in the following matrix forms

$$O_1 = \frac{1}{\omega\xi}\begin{pmatrix} \frac{1}{b^2d^2} & \frac{1}{bdac} \\ \frac{1}{bdac} & \frac{1}{a^2c^2} \end{pmatrix}, \qquad O_2 = \frac{1}{\omega\xi}\begin{pmatrix} \frac{1}{b^2d^2} & -\frac{1}{bdac} \\ -\frac{1}{bdac} & \frac{1}{a^2c^2} \end{pmatrix}, \qquad O_3 = \begin{pmatrix} A & 0 \\ 0 & B \end{pmatrix},$$

where

$$A = 1 - \frac{2}{\omega\xi b^2d^2}, \qquad B = 1 - \frac{2}{\omega\xi a^2c^2}.$$

Evidently, to let $O_3$ be a positive operator the parameter $\omega$ should be an appropriate value within the scope $1 \le \omega \le 2$, as is strongly relative to $a$, $b$, $c$ and $d$. Since

$$_{2m}\langle W|O_1|W\rangle_{2m} = {}_{2m}\langle W|O_2|W\rangle_{2m} = \frac{1}{4\omega\xi} \equiv p, \tag{20}$$

through the POVM measurement Bob can definitively get $|F\rangle_m$ or $|H\rangle_m$ with equal probability $p$. Alternatively, in terms of the POVM value Bob can positively conclude the state of qubit $m$. However, with the probability $1 - 2 \times p = 1 - \frac{1}{2\omega\xi}$ Bob may get $O_3$'s value. In this case, Bob can not infer which state the qubit $m$ is in. According to (18), after his POVM Bob knows exactly the state of his qubit 2 with probability $2p$. As a consequence, Bob can retrieve the original state $|P\rangle$ by performing an appropriate unitary operation. Explicitly, if Bob knows the state of his qubit 2 is $|E\rangle_2$ or $|G\rangle_2$, he needs only to perform the unitary operation $I_2 = |0\rangle_2\langle 0| + |1\rangle_2\langle 1|$ or $\sigma_2^z = |0\rangle_2\langle 0| - |1\rangle_2\langle 1|$, respectively. So far I have depicted the case that Charlie measures $|+\rangle_4$. As mentioned before Charlie may get $|-\rangle_4$. In the latter case, the QSTS process is trivially similar to that in the former one just proposed above. Alternatively, Bob can also restore the original state on his qubit 2 with the same probability. Thus, the total success probability of this tripartite QSTS scheme is

$$P = 4p = \frac{1}{\omega\xi} = \frac{(1-b^2)(1-d^2)b^2d^2}{1-b^2-d^2+2b^2d^2} \times \frac{1}{\omega}. \tag{21}$$

Above I have already shown the tripartite QSTS scheme of an arbitrary unknown single-qubit state. Now let me make some further discussions on it. As depicted previously, there are 8 possible results (see (5)–(12)) after Alice performing GHZ-basis measurement on her qubit pair $(x, 1, 3)$. It is possible that Alice gets one of other seven measurement results. For the other seven cases, applying the very similar analysis method just proposed above, the original state $|P\rangle$ can also be probabilistically recovered in either distant agent's place provided they collaborate with each other. Furthermore, I also work out the success probability for each case, and it is summarized detailedly in Table 1.

**Table 1** The relationship among Alice's measurement results (AM), the two operations of POVM ($O_i$) and the success probability of the QSTS (SP). More details can be seen in the text

| AM | $O_i$ | SP |
|---|---|---|
| $\lvert\lambda_1\rangle_{x13}$ ($\lvert\lambda_5\rangle_{x13}$) | $O_1=\frac{1}{\omega\xi}\begin{pmatrix}\frac{1}{a^2c^2} & \frac{1}{acbd}\\ \frac{1}{acbd} & \frac{1}{b^2d^2}\end{pmatrix}$, $O_2=\frac{1}{\omega\xi}\begin{pmatrix}\frac{1}{a^2c^2} & -\frac{1}{acbd}\\ -\frac{1}{acbd} & \frac{1}{b^2d^2}\end{pmatrix}$ | $\frac{(1-b^2)(1-d^2)b^2d^2}{1-b^2-d^2+2b^2d^2}\times\frac{1}{\omega}$ |
| $\lvert\lambda_2\rangle_{x13}$ ($\lvert\lambda_6\rangle_{x13}$) | $O_1=\frac{1}{\omega\xi}\begin{pmatrix}\frac{1}{a^2d^2} & \frac{1}{adbc}\\ \frac{1}{adbc} & \frac{1}{b^2c^2}\end{pmatrix}$, $O_2=\frac{1}{\omega\xi}\begin{pmatrix}\frac{1}{a^2d^2} & -\frac{1}{adbc}\\ -\frac{1}{adbc} & \frac{1}{b^2c^2}\end{pmatrix}$ | $\frac{(1-b^2)(1-d^2)b^2d^2}{b^2+d^2-2b^2d^2}\times\frac{1}{\omega}$ |
| $\lvert\lambda_3\rangle_{x13}$ ($\lvert\lambda_7\rangle_{x13}$) | $O_1=\frac{1}{\omega\xi}\begin{pmatrix}\frac{1}{b^2c^2} & \frac{1}{bcad}\\ \frac{1}{bcad} & \frac{1}{a^2d^2}\end{pmatrix}$, $O_2=\frac{1}{\omega\xi}\begin{pmatrix}\frac{1}{b^2c^2} & -\frac{1}{bcad}\\ -\frac{1}{bcad} & \frac{1}{a^2d^2}\end{pmatrix}$ | $\frac{(1-b^2)(1-d^2)b^2d^2}{b^2+d^2-2b^2d^2}\times\frac{1}{\omega}$ |
| $\lvert\lambda_4\rangle_{x13}$ ($\lvert\lambda_8\rangle_{x13}$) | $O_1=\frac{1}{\omega\xi}\begin{pmatrix}\frac{1}{b^2d^2} & \frac{1}{bdac}\\ \frac{1}{bdac} & \frac{1}{a^2c^2}\end{pmatrix}$, $O_2=\frac{1}{\omega\xi}\begin{pmatrix}\frac{1}{b^2d^2} & -\frac{1}{bdac}\\ -\frac{1}{bdac} & \frac{1}{a^2c^2}\end{pmatrix}$ | $\frac{(1-b^2)(1-d^2)b^2d^2}{1-b^2-d^2+2b^2d^2}\times\frac{1}{\omega}$ |

$O_3 = I - O_1 - O_2$

So the total success probability of this scheme is

$$4\times\frac{(1-b^2)(1-d^2)b^2d^2}{1-b^2-d^2+2b^2d^2}\times\frac{1}{\omega}+4\times\frac{(1-b^2)(1-d^2)b^2d^2}{b^2+d^2-2b^2d^2}\times\frac{1}{\omega}. \tag{22}$$

It can be easily verified that when all the coefficients $a$, $b$, $c$ and $d$ are $\frac{1}{\sqrt{2}}$ and $\omega = 1$, i.e., the states taken as the quantum channel are two Bell states and $O_3$ is a zero operator, the total success probability of QSTS will achieve 1. In this case, the present generalized QSTS scheme is transformed into the usual standard QSTS scheme.

## 3 Multiparty QSTS Scheme via POVM

Now let me generalize the tripartite scheme to a multiparty case. Suppose there are $N+1$ legitimate users. Alice is still the state sender, also the state she wants to split is given by (1). The other $N$ users are Alice's agents, named as $Bob_i, i = 1, 2, 3, \ldots, N$, respectively. Alice shares in advance with each agent a non-maximally entangled 2-qubit state

$$a_i\prod_{j=1}^{2}|U_j\rangle_{A_iB_i} + b_i\prod_{j=1}^{2}\left|U_j^c\right\rangle_{A_iB_i}, \quad i = 1, 2, 3, \ldots, N, \tag{23}$$

where $U_j$ is a binary variable $U_j \in \{0, 1\}$ and $U_j^c$ is its complement defined as $U_j^c = 1 - U_j$. Also $a_i$ and $b_i$ are nonzero real numbers, and they satisfy $|a_i| \geq |b_i|$ and $|a_i|^2 + |b_i|^2 = 1$ ($i = 1, 2, 3, \ldots, N$), respectively. All qubits $A_i$ ($i = 1, 2, 3, \ldots, N$) belong to Alice and qubit $B_i$ is in the $i$th agent's place. Without loss of generality, suppose this quantum channel have the same form as that proposed above. Then the state of the system can be simplified as

$$|\Psi\rangle = \prod_{i=1}^{N}(a_i|00\rangle_{A_iB_i} + b_i|11\rangle_{A_iB_i}) \otimes (\alpha|0\rangle + \beta|1\rangle). \tag{24}$$

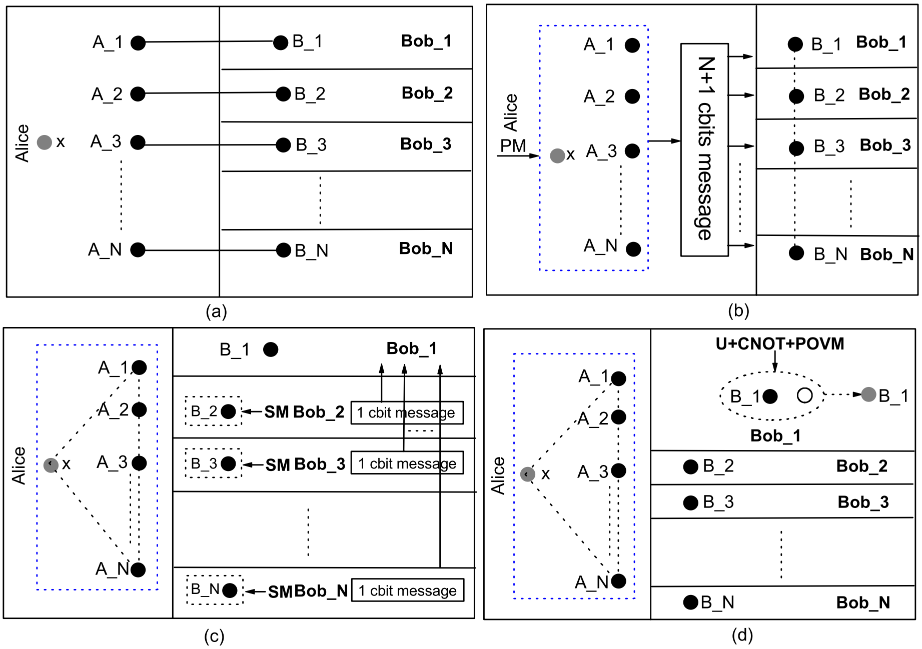


**Fig. 2** Partition of an unknown arbitrary single-qubit state among $N$ recipients by using $N$ non-maximally entangled 2-qubit states. The *solid dots* represent qubits, the *litter hollow circle* stands for the auxiliary qubit. The *dotted rectangle* characterizes $(N+1)$-qubit GHZ-basis projective measurement (PM). SM symbolizes a single-qubit measurement. CNOT means the controlled-not gate operation and POVM represents the positive operator-valued measure. Also see text for more details

Similarly, to split her unknown state between the $N$ agents $Bob_i, i = 1, 2, 3, \ldots, N$ by utilizing the quantum channels, Alice performs an $(N+1)$-qubit GHZ-basis measurement on her $N+1$ qubits ($x$ is numbered $N+1$). The measurement is such that it projects the $N+1$ qubits to the $(N+1)$-qubit (spin-1/2) complete orthonormal basis

$$|\lambda'\rangle = \prod_{n}^{N+1} |V_n\rangle + (-1)^h \prod_{n}^{N+1} |V_n^c\rangle, \tag{25}$$

where $V_n \in \{0, 1\}$, $V_n^c = 1 - V_n$, $h \in \{0, 1\}$. After Alice's measurement, the state of the rest $N$ qubits collapse to a pure entangled state which contains all the information of the state $|P\rangle$. In other word, the distribution of the quantum information is completed. Then to restore the original state, Alice declares her measurement result via a classical channel. Due to symmetry, anyone of the $N$ agents can restore the original state with others' assistance. Without loss of generality, suppose $Bob_1$ is designed to restore the state $|P\rangle$. Then the other $(N-1)$ agents are asked to measure their respective qubits in the $X$ bases. Similarly, provided that $Bob_1$ gets the other $N-1$ agents' measurement results, he can realize the multiparty QSTS probabilistically with the same steps proposed earlier. I would not depict it anymore hereafter, and the whole process is amply illustrated in Fig. 2. Intuitively, the QSTS process in the multiparty case can also be realized in a deterministic manner when the states taken as the quantum channels are Bell state and $\omega$ is chosen as 1.

## 4 Remark and Summary

The security of this protocol against eavesdropping and cheating can be assured by using the same check and proof methods proposed in Refs. [9, 20, 39, 40, 43]: any eavesdropping leads to the discrepancy between the state that Alice sends and the state that legitimate user reconstructs. Thus eavesdropping can be detected by publicly comparing a subset of the quantum states.

To summarize, in this paper I have explicitly presented a symmetric and probabilistic QSTS scheme for sharing an arbitrary unknown single-qubit state. In the scheme the quantum channel employed by the involved parties is two non-maximally entangled 2-qubit states. To realize quantum state sharing, the state sender performs a GHZ-basis projective measurement and publish the measurement result via a classical channel. Then one agent Bob is designated to restore the original state while another one acts as an assistant. By collaboration, the two agents can reconstruct probabilistically the original state by incorporating an auxiliary qubit and executing appropriate operations including a proper POVM. Moreover, I have also worked out the success probability of the scheme. At last, I sketch the generalization of the tripartite scheme to a multiparty case.

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